Reflection and Transmission Characteristics of an Uncoincidental Junction on Rectangular Dielectric Waveguides

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***Abstract*—A novel approach to the analysis of an uncoincidental junction in rectangular dielectric waveguide (RDWG) is presented. The boundary condition equations on the junction plane are presented through the tangential components of guided modes and radiation modes which are excited at the discontinuity. To obtain the reflection and transmission coefficients at the discontinuity, the equations are transformed into the spectral domain by the two-dimensional Fourier transformation. In the spectral domain, the orthogonal relationship between the guided and radiation modes is used to determine the reflection and transmission coefficients approximately. The parameters corresponding to the phase constant of the radiation fields in the spectral domain are defined by using an iterative calculation. The reflection and transmission coefficients are determined exactly. These coefficients are compared with 10 GHz band experiments. Our analysis is found to be in good agreement with experimental results.**

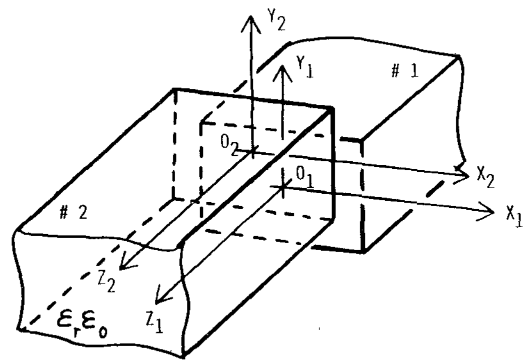

Fig. 1. Uncoincidental junction of rectangular dielectric waveguide and coordinate system.

I. Introduction

DIELECTRIC waveguide which has a rectangular cross section is an important element in integrated circuits for millimeter, submillimeter, and optical wave ranges. It is necessary to know the reflection, transmission, and radiation characteristics at discontinuities on the guides when the integrated circuit is designed. In previous work, these problems have for the most part been treated as coupling problems between solid-state lasers and slab waveguides.

Rozzi [1] presented a variational treatment for the diffraction of TE and TM waves on the discontinuity and solved the problem by the Ritz–Galerkin approach. The guide discontinuity was analyzed by means of the least-squares boundary residual method [2].

On the other hand, Hockham [3] solved the problems of solid-state laser radiation by introducing the radiation mode, which is expressed in spectral form. Lewin [4], [5] analyzed a similar problem by taking into account the reflected guided mode components which Hockham had neglected.

Takenaka *et al.* [6] improved upon Lewin's method. They transformed the boundary condition equations at the slab waveguide junction into the spectral domain and, using an iterative method, calculated the ratio of the electric to the magnetic field on the junction more precisely.

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We extended Takenaka's method to three-dimensional problems and analyzed the characteristics of an abruptly ended rectangular dielectric waveguide (RDWG) [7] and of a coaxial junction of different sizes of RDWG [8].

In this paper, we present the reflection and transmission characteristics of an uncoincidental junction of a RDWG by applying the analytical method in [7] and [8].

Outlines of the analytical approaches are as follows. (1) The electromagnetic field on the uncoincidental junction plane is expressed by guided and radiation mode fields. (2) A pair of field equations which must be satisfied with boundary conditions on the junction plane is defined. (3) The two-dimensional Fourier transformation is introduced and applied to these equations. (4) These equations are solved to obtain the approximate radiation mode field by introducing the phase constant of the radiation mode. (5) Considering the orthogonal relations between guided and radiation modes, these phase constants are made to converge to a constant value by means of an iterative method. Then, the reflection and transmission coefficients are also converged to final values.

II. Analysis

A. Field Expression on Junction Plane

An uncoincidental junction in a RDWG with dielectric constant $\epsilon_r\epsilon_0$ and coordinate system are shown in Fig. 1.

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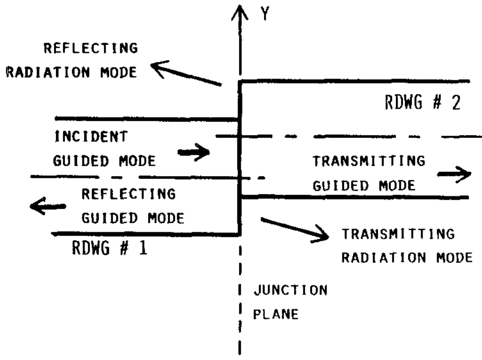

Fig. 2. Field components around the uncoincidental junction.

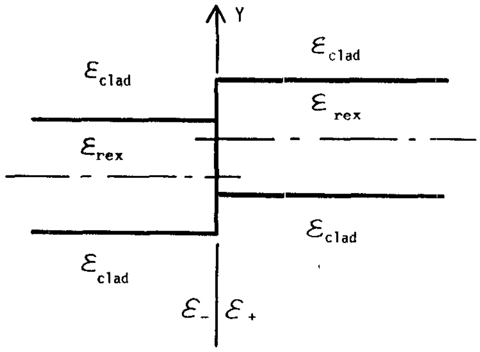

Fig. 3. Distribution of the dielectric constant around the uncoincidental junction.

Waveguides #1 and #2 are assumed to be single-mode waveguides and to have the same dimensions. The fundamental E_{11}^{y} mode propagates on waveguide #1 from negative infinity of z. An approximated field expression of the guided mode (E_{11}^{y}) has been given in [9, appendix]. Two guides are joined uncoincidentally at $z=0$. The surrounding region of the guide has a dielectric constant ϵ_c and a permeability μ_0. As the main field components of the E_{11}^{y} mode are E_y and H_x, we concentrate our attention on them. Referring to Fig. 2, we obtain the following boundary condition equations:

$$\begin{aligned} E_y^i(x,y) &+ R_0 E_y^i(x,y) + \int_0^\infty \int_0^\infty R(\rho_x,\rho_y) \\ &\times E_y^t(\rho_x,\rho_y,x,y)\,d\rho_x\,d\rho_y \\ &= T_0 E_y^t(x,y) \\ &+ \int_0^\infty \int_0^\infty T(\rho_x,\rho_y) E_y^t(\rho_x,\rho_y,x,y)\,d\rho_x\,d\rho_y \end{aligned} \tag{1}$$

$$\begin{aligned} H_x^i(x,y) &- R_0 H_x^i(x,y) \\ &- \int_0^\infty \int_0^\infty R(\rho_x,\rho_y) H_x^t(\rho_x,\rho_y,x,y)\,d\rho_x\,d\rho_y \\ &= T_0 H_x^t(x,y) \\ &+ \int_0^\infty \int_0^\infty T(\rho_x,\rho_y) H_x^t(\rho_x,\rho_y,x,y)\,d\rho_x\,d\rho_y. \end{aligned} \tag{2}$$

In the above equations, $E_y^i(x,y)$ and $H_x^i(x,y)$ are the electric and magnetic fields of the incident guided modes. R_0 and T_0 are the reflection and transmission coefficients of the guided mode fields. $E_y^t(\rho_x,\rho_y,x,y)$ and $H_x^t(\rho_x,\rho_y,x,y)$ are the electric and magnetic field components of the transmitted radiation mode, respectively. $R(\rho_x,\rho_y)$ and $T(\rho_x,\rho_y)$ are the reflection and transmission coefficients of the radiation mode fields. The quantities ρ_x and ρ_y are transverse phase constants of the radiation modes. In the above expression, the superscripts i and t indicate the incident and transmitted components. The superscript r, which will appear in (4) through (7), indicates the reflected component.

The two-dimensional Fourier transformation (TDFT) is introduced to put (1) and (2) into the spectral form:

$$G(\mu,\nu) = \frac{1}{4\pi^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} g(x,y) e^{j(\mu x - \nu y)}\,dx\,dy \tag{3}$$

where μ and ν are the spectral variables.

Substituting (3) into (1) and (2), we obtain the following boundary conditions in the spectral domain:

$$\begin{aligned} (1+R_0)e^i(\mu,\nu) &+ \mathscr{E}^r(\mu,\nu) \\ &= T_0 e^t(\mu,\nu) + \mathscr{E}^t(\mu,\nu) \end{aligned} \tag{4}$$

$$\begin{aligned} (1-R_0)h^i(\mu,\nu) &+ \mathscr{H}^r(\mu,\nu) \\ &= T_0 h^t(\mu,\nu) + \mathscr{H}^t(\mu,\nu) \end{aligned} \tag{5}$$

where $e^i(\mu,\nu)$ and $h^i(\mu,\nu)$ are spectral expressions of the electric and magnetic fields of the guided mode. In the above equations, the effects of the shift of guide axes in both the x and the y direction on the discontinuity plane are taken into account:

$$e^t(\mu,\nu) \propto e^i(\mu,\nu) e^{j(\mu c - \nu d)}$$

$$h^t(\mu,\nu) \propto h^i(\mu,\nu) e^{j(\mu c - \nu d)}$$

where c and d indicate shifting quantities of the x and y axes, respectively. The quantities $\mathscr{E}^{r,t}(\mu,\nu)$ and $\mathscr{H}^{rt}(\mu,\nu)$ are spectral expressions of the radiation mode fields, which were shown in integral form in (1) and (2).

As the field components expressed in the spectral domain should be satisfied with Maxwell's equation, the following relations can be obtained for the front and back sides of the junction:

$$h^r(\mu,\nu) = \frac{\omega\epsilon_0\epsilon_-}{\beta} e^r(\mu,\nu) \qquad \text{for } z<0 \tag{6}$$

$$h^t(\mu,\nu) = -\frac{\omega\epsilon_0\epsilon_+}{\beta} e^t(\mu,\nu) \qquad \text{for } z>0 \tag{7}$$

where β is a phase constant of the guided mode in both guides #1 and #2. The quantities ϵ_- and ϵ_+ represent the relative dielectric constant distribution in the back and front regions of the junction as shown in Fig. 3 [9].

From (6) and (7), the following relations are assumed to be kept for the radiation modes:

$$\mathscr{H}^r(\mu,\nu) = \frac{\omega\epsilon_0\epsilon_-}{p(\mu,\nu)} \mathscr{E}^r(\mu,\nu) \tag{8}$$

$$\mathscr{H}^t(\mu,\nu) = -\frac{\omega\epsilon_0\epsilon_+}{q(\mu,\nu)} \mathscr{E}^t(\mu,\nu). \tag{9}$$

In the above equations, the parameters $p(\mu,\nu)$ and $q(\mu,\nu)$ have the following physical meaning: $p(\mu,\nu)$ is the phase constant of radiation modes in the spectral domain for the $z<0$ region (RDWG #1 side), and $q\ (\mu,\nu)$ is the constant for the region of $z>0$ (#2 side).

Since $p(\mu,\nu)$ and $q(\mu,\nu)$ cannot be defined exactly at this stage, we use the following expression as the zeroth-order approximation [7], [8]:

$$p(\mu,\nu) = q(\mu,\nu) = \begin{cases} \sqrt{\epsilon_c k_0^2 - \mu^2 - \nu^2}, & \epsilon_c k_0^2 \geqslant \mu^2+\nu^2 \\ -j\sqrt{\mu^2+\nu^2-\epsilon_c k_0^2}, & \epsilon_c k_0^2 < \mu^2+\nu^2 \end{cases} \tag{10}$$

$$k_0^2 = \omega^2 \mu_0 \epsilon_0.$$

The above-mentioned expression means that $p(\mu,\nu)$ and $q(\mu,\nu)$ are the phase constants in the spectral domain of a plane wave in a homogeneous medium with dielectric constant [5].

As the cores of waveguides #1 and #2 occupy a very small region in the surrounding media, we may use (10) as the zeroth-order approximation. The constants $p(\mu,\nu)$ and $q(\mu,\nu)$ are made to converge to constant values by an iterative approach, as shown below.

B. Derivation of the Reflection Coefficient R_0 *and Transmission Coefficient* T_0

As a result of substituting (8) and (9) into (4) and (5), the spectral forms of the reflected and transmitted radiation electric fields can be obtained as follows:

$$\mathscr{E}^r(\mu,\nu) = \frac{1}{\omega\epsilon_0\left\{\dfrac{\epsilon_+}{q(\mu,\nu)} + \dfrac{\epsilon_-}{P(\mu,\nu)}\right\}} \cdot \left[(R_0-1)h^t(\mu,\nu) - (1+R_0)e^t(\mu,\nu)\frac{\omega\epsilon_0\epsilon_-}{q(\mu,\nu)} + \left\{\frac{\omega\epsilon_0\epsilon_+}{q(\mu,\nu)}e^t(\mu,\nu) + h^t(\mu,\nu)\right\}T_0\right] \tag{11}$$

$$\mathscr{E}^t(\mu,\nu) = \frac{1}{\omega\epsilon_0\left\{\dfrac{\epsilon_+}{q(\mu,\nu)} + \dfrac{\epsilon_-}{P(\mu,\nu)}\right\}} \cdot \left[(R_0+1)\frac{\omega\epsilon_0\epsilon_-}{P(\mu,\nu)} - (1-R_0)h^t(\mu,\nu) + \left\{h^t(\mu,\nu) - \frac{\omega\epsilon_0\epsilon_-}{p(\mu,\nu)}e^t(\mu,\nu)\right\}T_0\right]. \tag{12}$$

These expressions show that the electric field of the radiation mode in the spectral domain can be expressed as a function of the field distribution of the guided modes e^t and h^t, the reflection and transmission coefficients R_0 and T_0, and the parameters $p(\mu,\nu)$ and $q(\mu,\nu)$. However, in this step, the correct values of $R_0, T_0, p(\mu,\nu)$, and $q(\mu,\nu)$ are not yet known.

To determine R_0 and T_0, an orthogonal relationship between radiation and guided modes is utilized, and the following procedures with the recurrence relation for $p(\mu,\nu)$ and $q(\mu,\nu)$ are introduced.

Step 1: We introduce the orthogonal relation, denoted Δ_1 and Δ_2, between the electric field of the radiation mode and the magnetic field of the guided mode as follows. For $z<0$ (in the region of RDWG #1),

$$\Delta_1 = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \mathscr{E}^r(\mu,\nu) h^{r*}(\mu,\nu)\, d\mu\, d\nu \tag{13}$$

while for $z>0$ (in the region of RDWG #2),

$$\Delta_2 = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \mathscr{E}^t(\mu,\nu) h^{t*}(\mu,\nu)\, d\mu\, d\nu \tag{14}$$

where the $*$ denotes the complex conjugate of a function.

If the parameters $p(\mu,\nu)$ and $q(\mu,\nu)$ are defined correctly, R_0 and T_0 can also be derived correctly by setting (13) and (14) equal to zero. Unfortunately, (13) and (14) are not equal to zero, because $p(\mu,\nu)$ and $q(\mu,\nu)$ in this step are given approximately by (10).

Step 2: We start with an iterative calculation for R_0 and T_0 by setting (13) and (14) equal to zero. As a result, we can find a zeroth-order approximated expression for R_0 and T_0 as follows:

$$R_0 = \frac{\iint_{-\infty}^{\infty} A_r B_{qt}^+\, d\mu\, d\nu \cdot \iint_{-\infty}^{\infty} A_t B_{pt}^-\, d\mu\, d\nu - \iint_{-\infty}^{\infty} A_r B_{qt}^+\, d\mu\, d\nu \cdot \iint_{-\infty}^{\infty} A_t B_{pt}^-\, d\mu\, d\nu}{\iint_{-\infty}^{\infty} A_r B_{qt}^-\, d\mu\, d\nu \cdot \iint_{-\infty}^{\infty} A_t B_{pt}^-\, d\mu\, d\nu - \iint_{-\infty}^{\infty} A_r B_{qt}^+\, d\mu\, d\nu \cdot \iint_{-\infty}^{\infty} A_t B_{pt}^+\, d\mu\, d\nu} \tag{15}$$

$$T_0 = \frac{\iint_{-\infty}^{\infty} A_r B_{qt}^-\, d\mu\, d\nu \cdot \iint_{-\infty}^{\infty} A_t B_{pt}^-\, d\mu\, d\nu - \iint_{-\infty}^{\infty} A_r B_{qt}^+\, d\mu\, d\nu \cdot \iint_{-\infty}^{\infty} A_t B_{pt}^+\, d\mu\, d\nu}{\iint_{-\infty}^{\infty} A_r B_{qt}^-\, d\mu\, d\nu \cdot \iint_{-\infty}^{\infty} A_t B_{pt}^-\, d\mu\, d\nu - \iint_{-\infty}^{\infty} A_r B_{qt}^+\, d\mu\, d\nu \cdot \iint_{-\infty}^{\infty} A_t B_{pt}^+\, d\mu\, d\nu} \tag{16}$$

where

$$A_r = \frac{h^{r*}(\mu,\nu)}{\dfrac{\epsilon_+}{q(\mu,\nu)} + \dfrac{\epsilon_-}{p(\mu,\nu)}}$$

$$A_t = \frac{h^{t*}(\mu,\nu)}{\dfrac{\epsilon_+}{q(\mu,\nu)} + \dfrac{\epsilon_-}{p(\mu,\nu)}}$$

$$B_{\alpha,m}^{\pm} = h^m(\mu,\nu) \pm \frac{\omega\epsilon_0\epsilon_\mp}{\alpha} e^m(\mu,\nu)$$

$$\alpha = p(\mu,\nu) \quad \text{or} \quad q(\mu,\nu) \qquad m = i \text{ or } t.$$

R_0 and T_0 can be calculated by means of a numerical method, and they are referred to in this step as R_0' and T_0'.

Step 3: As in step 1, two other parameters, $\tilde{\Delta}_1$ and $\tilde{\Delta}_2$, are introduced in terms of the magnetic field of the radiation mode and the electric field of the guided mode.

For $z < 0$ (in the region of RDWG #1),

$$\tilde{\Delta}_1 = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^r(\mu,\nu)\left\{\frac{\omega\epsilon_0\epsilon_-}{p(\mu,\nu)}\mathscr{E}^r(\mu,\nu)\right\}^* d\mu\, d\nu \quad (17)$$

while for $z > 0$ (in the region of RDWG #2),

$$\tilde{\Delta}_2 = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \mathscr{E}^t(\mu,\nu)\left\{-\frac{\omega\epsilon_0\epsilon_+}{q(\mu,\nu)}\mathscr{E}^t(\mu,\nu)\right\}^* d\mu\, d\nu. \quad (18)$$

In this step $\tilde{\Delta}_1$ and $\tilde{\Delta}_2$ are not yet equal to zero owing to the use of the approximated $p(\mu,\nu)$ and $q(\mu,\nu)$.

Step 4: Procedures similar to those shown in step 2 are applied to $\tilde{\Delta}_1$ and $\tilde{\Delta}_2$. Then expressions for R_0 and T_0 can be obtained as follows:

$$R_0 = \frac{\iint_{-\infty}^{\infty} C_r B_{qi}^+ \,d\mu\, d\nu \cdot \iint_{-\infty}^{\infty} C_t B_{pt}^- \,d\mu\, d\nu - \iint_{-\infty}^{\infty} C_r B_{qt}^+ \,d\mu\, d\nu \cdot \iint_{-\infty}^{\infty} C_t B_{pi}^- \,d\mu\, d\nu}{\iint_{-\infty}^{\infty} C_r B_{qi}^- \,d\mu\, d\nu \cdot \iint_{-\infty}^{\infty} C_t B_{pt}^- \,d\mu\, d\nu - \iint_{-\infty}^{\infty} C_r B_{qt}^+ \,d\mu\, d\nu \cdot \iint_{-\infty}^{\infty} C_t B_{pi}^+ \,d\mu\, d\nu} \quad (19)$$

$$T_0 = \frac{\iint_{-\infty}^{\infty} C_r B_{qi}^- \,d\mu\, d\nu \cdot \iint_{-\infty}^{\infty} C_t B_{pi}^- \,d\mu\, d\nu - \iint_{-\infty}^{\infty} C_r B_{qi}^+ \,d\mu\, d\nu \cdot \iint_{-\infty}^{\infty} C_t B_{pi}^+ \,d\mu\, d\nu}{\iint_{-\infty}^{\infty} C_r B_{qi}^- \,d\mu\, d\nu \cdot \iint_{-\infty}^{\infty} C_t B_{pt}^- \,d\mu\, d\nu - \iint_{-\infty}^{\infty} C_r B_{qt}^+ \,d\mu\, d\nu \cdot \iint_{-\infty}^{\infty} C_t B_{pi}^+ \,d\mu d\nu} \quad (20)$$

where

$$C_t = \frac{\epsilon_+ e^{t*}(\mu,\nu)}{\dfrac{q(\mu,\nu)}{p(\mu,\nu)}\epsilon_- + \epsilon_+} \qquad C_r = \frac{\epsilon_- e^{r*}(\mu,\nu)}{\dfrac{p(\mu,\nu)}{q(\mu,\nu)}\epsilon_- + \epsilon_+}.$$

R_0 and T_0 as calculated here are written as $\tilde{R}_0$ and $\tilde{T}_0$.

Step 5: Comparing the values of R_0' and $\tilde{R}_0$ (T_0' and $\tilde{T}_0$), we must decide whether the calculation procedure is continued or not.

(i) When $|R_0' - \tilde{R}_0|$ ($|T_0' - \tilde{T}_0|$) is smaller than a criterion (δ), which can be determined by the accuracy necessary for the calculation, the final result of R_0 (T_0) is obtained as $R_0 = R_0' \simeq \tilde{R}_0$ ($T_0 = T_0' \simeq \tilde{T}_0$).

(ii) When $|R_0' - \tilde{R}_0|$ or $|T_0' - \tilde{T}_0|$ is larger than that criterion, step 6 should be followed.

Step 6: Approximated recurrence relations for $p(\mu,\nu)$ and $q(\mu,\nu)$ are introduced as follows [7], [8]:

$$\frac{1}{p_{(n+1)}(\mu,\nu)} = \frac{1}{p_{(n)}(\mu,\nu)} - 4\pi^2\frac{\tilde{\Delta}_{1(n)}h^r(\mu,\nu)}{\omega\epsilon_0\epsilon_-\epsilon^r(\mu,\nu)} \quad (21)$$

$$\frac{1}{q_{(n+1)}(\mu,\nu)} = \frac{1}{q_{(n)}(\mu,\nu)} - 4\pi^2\frac{\tilde{\Delta}_{2(n)}h^t(\mu,\nu)}{\omega\epsilon_0\epsilon_+\epsilon^t(\mu,\nu)}. \quad (22)$$

In the above equations, it is assumed that the $(n+1)$th field expression is not so different from the (n)th field, and the incident power of the guided mode in the RDWG #1 is normalized to 1 W.

Until $|R_0' - \tilde{R}_0|$, ($|T_0' - \tilde{T}_0|$) become smaller than δ, steps 1 through 6 are iterated by substituting (21) and (22) into (11) and (12).

The above-mentioned method is considered to give an approximate value of R_0 and T_0 under the condition whereby $\tilde{\Delta}_1$ and $\tilde{\Delta}_2$ are sufficiently small; i.e., the orthogonal relation between the radiation and the guided mode is approximately satisfied.

III. Numerical Calculations

A. Field Expression of Guided Mode on the Rectangular Dielectric Waveguide

For calculating the reflection and transmission coefficients, it is necessary to determine the transverse field expression of the guided modes. However, a rigorous field expression for a rectangular dielectric waveguide has not been found. In this paper, an approximated field expression of the guided mode is introduced and used. The transverse plane on the waveguide is divided into nine subregions (as shown in Fig. 4) and the field distribution

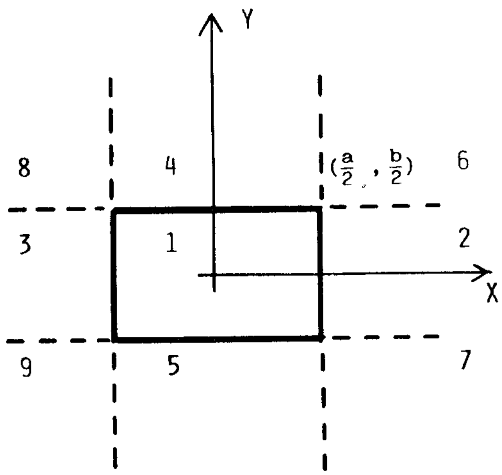

Fig. 4. Nine subregions for field expression of guided mode on the rectangular dielectric waveguide.

in each region is defined explicitly:

$$E_y(x, y) = -\frac{\beta}{\omega\epsilon_0\epsilon_{rex}} A\cos(k_x x)\cos(k_y y) \quad \text{in region 1} \tag{23}$$

$$E_y(x, y) = -\frac{\beta}{\omega\epsilon_0\epsilon_{rex}} A\cos\left(k_x\frac{a}{2}\right)\cos(k_y y)\, e^{-\gamma_x(|x|-a/2)} \quad \text{in regions 2 and 3} \tag{24}$$

$$E_y(x, y) = -\frac{\beta}{\omega\epsilon_0\epsilon_c} A\cos(k_x x)\cos\left(k_y\frac{b}{2}\right) e^{-\gamma_y(|y|-b/2)} \quad \text{in regions 4 and 5} \tag{25}$$

$$E_y(x, y) = -\frac{\beta}{\omega\epsilon_0\epsilon_c} A\cos\left(k_x\frac{a}{2}\right)\cos\left(k_y\frac{b}{2}\right) \cdot e^{-\gamma_x(|x|-a/2)} e^{-\gamma_y(|y|-b/2)} \quad \text{in regions 6, 7, 8, and 9.} \tag{26}$$

B. Calculation Results

Three types of uncoincidental junction are shown in Fig. 5. Guide axes of type I are displaced by c only in the y direction. Axes of type II are displaced by d only in the x direction. Axes of type III are displaced by c and d in both the x and the y direction. The calculation of R_0 and T_0 has been carried out with the following dimensions: relative dielectric constant of both waveguides $\epsilon_r = 2.01$; cross section of the guide, 20×20 mm; frequency, 10 GHz.

An iterative calculation is carried out until the values of $\tilde{\Delta}_1$ and $\tilde{\Delta}_2$ are reduced to values smaller than a criterion of 10^{-6}. Then the resulting R_0 and T_0 can be seen to converge to steady values. In this example, it requires four iterative calculations to converge R_0 and T_0 for type I and two iterations for type II.

Calculated results for uncoincidental displacement value versus $|R_0|^2$, $|T_0|^2$, and P_r are shown in Figs. 6 to 8. In these figures, the input power of the guided mode is normalized to 1 W. Dotted lines show the experimental results, which are described in the next section.

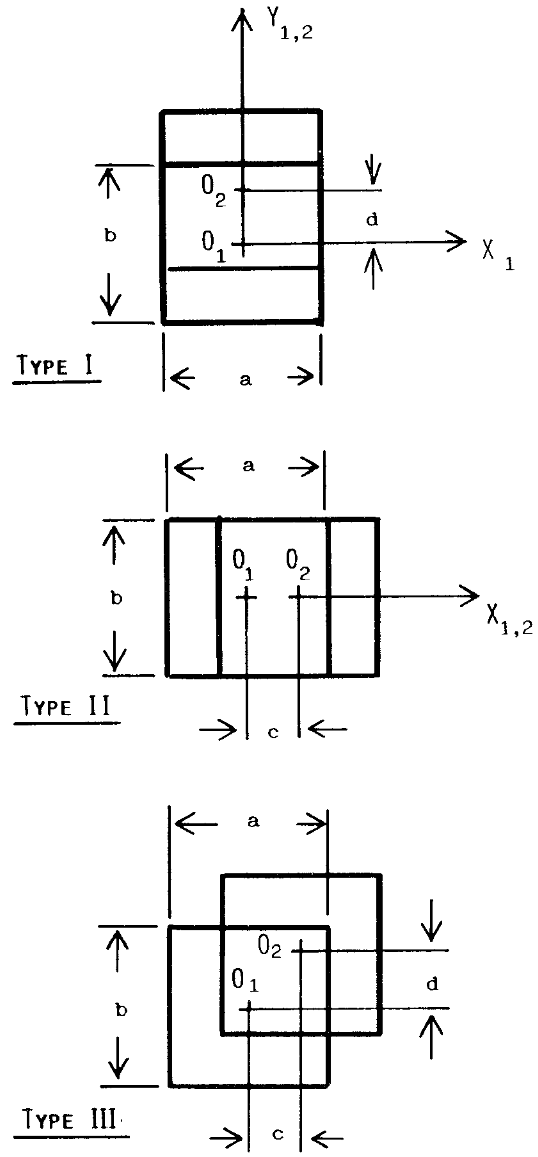

Fig. 5. Three types of the uncoincidental junction.

The total radiation power P_r from the uncoincidental junction can be obtained from the following relation:

$$P_r = \operatorname{Re}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \mathscr{E}^r(\mu, \nu)\cdot\mathscr{H}^{r*}(\mu, \nu)\, d\mu\, d\nu + \operatorname{Re}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \mathscr{E}^t(\mu, \nu)\cdot\mathscr{H}^{t*}(\mu, \nu)\, d\mu\, d\nu. \tag{27}$$

The first term on the right-hand side shows the component of backward radiation power and the second term is the forward radiation power.

From the principle of energy conservation, the sum of $|R_0|^2$, $|T_0|^2$, and P_r should be equal to unity. But in our calculations, the discrepancy between this sum and unity is less than 0.0003.

IV. Experiments and Discussions

Fig. 9 shows an experimental setup to measure the transmission coefficient of the guided mode on the uncoincidental junction of rectangular dielectric waveguides. On

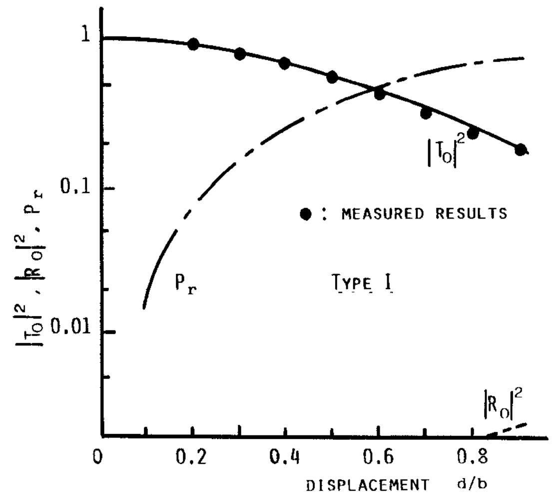

Fig. 6. Axial displacement versus reflection, transmission coefficient R_0, T_0, and radiation power P_r on the uncoincidental junction for x displacement, type I.

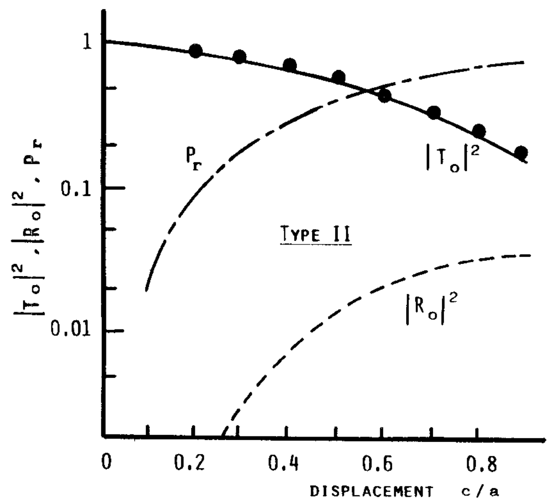

Fig. 7. Same as Fig. 6 for the y displacement, type II.

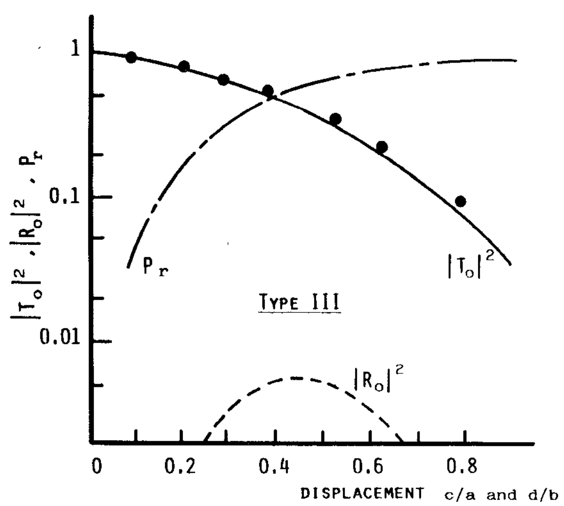

Fig. 8. Same as Fig. 6 for the x and y displacement, type III.

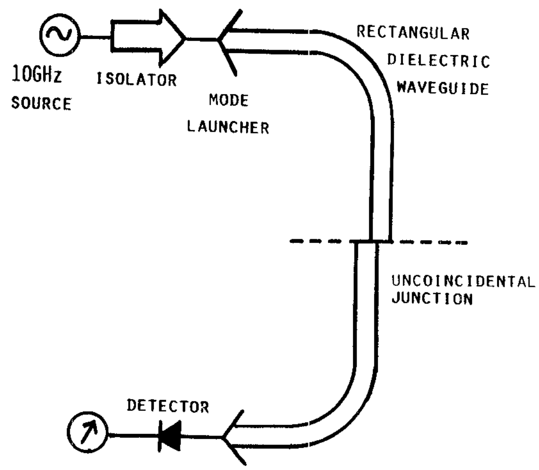

Fig. 9. Experimental setup.

the junction plane, two guides (#1 and #2) are jointed uncoincidentally with an x and/or y displacement. The displacement value can be varied precisely with a guide mounting mechanism. Both waveguides have 20 mm×20 mm cross sections, a length of approximately 1 m, and a relative dielectric constant of 2.01. The E_y^{11} mode of 10 GHz is launched into waveguide #1 with a mode launcher. The wave is transmitted through waveguide #2 over the junction and finally reaches the detector.

Measured results of the transmission coefficient (T_0) are shown in Figs. 6 to 8 as dotted marks. The results are found to be in good agreement with the calculated results.

As the incident E_y^{11} mode has a y-directed polarization, the uncoincidental junction in which the guide axes are displaced in the y direction (as type I in Fig. 5) is referred to as the y displacement. On the contrary, type II is referred to as the x displacement.

From the above studies, a relationship between the direction of displacement and R_0, T_0, and P_r can be found as follows:

1) On the transmission coefficient T_0, the y displacement gives a value about twice that of the x displacement for the same displacement amount.
2) On the reflection coefficient R_0, the x displacement gives a value about ten times greater than that of the same amount of the y displacement.
3) Total radiation power P_r from the junction of the x displacement is about 1.1 times that of the y displacement.

V. Conclusions

Reflection, transmission, and radiation characteristics of the uncoincidental junction of rectangular dielectric waveguide are studied by a newly introduced analytical method and by experiments. As a result it is found that the uncoincidental junction in which the waveguide axes are displaced in the y direction, that is, the same direction as the electric field polarization of the guided mode, gives a relatively small reflection coefficient R_0 and a larger T_0

than for an x displacement, even though both displacements are of the same amount.

The characteristics for the case where the guide axes are displaced in the c to x and d to y directions are also obtained. The characteristics for this case have never been obtained from the previous dielectric slab approximation.

The analytical method and results of this study will be useful in analyzing the rectangular dielectric waveguide junction for integrated circuits in the millimeter through optical wave region and in determining the allowance for the uncoincidental displacement of the guides.

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